

PART - A
Answer ALL questions

1. What is meant by generalized coordinates?
2. Write the Lagrangian for a free particle in cylindrical coordinates.
3. Show that the kinetic energy T for a torque free motion of rigid body is a constant.
4. When $\mathbf{L}=\mathbf{r} \times \mathbf{p}$ and $\mathbf{N}=\mathbf{r} \times \mathbf{F}$, show that $\mathbf{N}=\mathrm{d} \mathbf{L} / \mathrm{dt}$

05 . What is a Coroilis force ? Give one example.
06. State the principle of least action.
07. Show that the generating function $\mathrm{F}_{1}=\mathrm{q} Q$ generates a transformation that interchanges the momenta and coordinates.
08. Show that $\left[\mathrm{q}_{\mathrm{i}}, \mathrm{p}_{\mathrm{j}}\right]_{\mathrm{q}, \mathrm{p}}=0$ for $\mathrm{i} \neq \mathrm{j}$ and 1 for $\mathrm{i}=\mathrm{j}$.
09. Show that the Hamilton's principal function $S$ differs from the indefinite time integral of the Lagrangian by a constant.
10. What is meant by secular equation?

## PART - B

Answer any FOUR questions
11. Show that the charged particle in an electromagnetic field has a potential $\mathrm{U}=\mathrm{q} \phi-\mathrm{qA.v}$
12. Solve the Euler's equations of motion for a symmetric top $\mathrm{I}_{1}=\mathrm{I}_{2} \neq \mathrm{I}_{3}$ with no torque acting on it.
13. Using the basic definition of the Hamiltonian $\mathrm{H}(\mathrm{q}, \mathrm{p}, \mathrm{t})$, obtain Hamilton's canonical equations of motion
14. Show that the transformation $\mathrm{Q}=\mathrm{q}+\mathrm{i} \mathrm{p}$ and $\mathrm{P}=\mathrm{Q}-\mathrm{i} \mathrm{p}$ is not canonical. Suppose the size of the units used to measure the coordinates and momentum are changed to $\mathrm{Q}^{\prime}$ and $\mathrm{P}^{\prime}$ such that $\mathrm{Q}^{\prime}=\mu \mathrm{Q}$ and $\mathrm{P}^{\prime}=v \mathrm{P}$ then show the transformation equations are canonical.
15. Solve by the Hamilton-Jacobi method the motion of a particle in a plane under the action of a central potential $\mathrm{V}(\mathrm{r})$ to obtain the equation of orbit.

PART - C
Answer any FOUR questions
$(4 \times 12.5=50)$
16. a) A particle of mass $m$ is attached to the mid point of a weightless rod of length $L$. The ends of the rod are constrained to move along the x and y axes without friction. Write the Lagrangian and solve for the equation of motion assuming $g$ acts in the negative $y$ direction. (7.5)
b) A mass m is attached to a spring of stiffness constant k and capable of motion along the x direction. Using Hamilton's canonical equations find the equation of motion for the mass. (5)
17. a) Show that $\mathrm{Q}=\log (\sin \mathrm{p}) / \mathrm{q}$ and $\mathrm{P}=\mathrm{q} \cot \mathrm{p}$ is canonical using Hamilton's canonical equations. (7.5)
b. Show that the transformation given by $2 P=p^{2}+q^{2}$ and $Q=\tan ^{-1} q / p$ is canonical.
18. Set up the Hamiltonian for the one dimensional harmonic oscillator and using the method of separation of variables evaluate $S$ and hence obtain the solution for the oscillators $(2 \alpha / \mathrm{k})^{1 / 2} \cos \omega(\mathrm{t}+\beta)$. Using the initial conditions at $t=0 q=q_{0}, p=p_{0}$ and $\beta=0$ prove that $S=\int L d t$ for the linear harmonic oscillator.
19. Set up the Lagrangian for the linear triatomic molecule and solve for the normal modes of vibrations.
20. Write notes on any Two of the following
i) Lagrange's equation from the variational principle. ii)Theory of Hamilton -Jacobi method.
iii) Invariance of Poisson's brackets in a canonical transformation.

